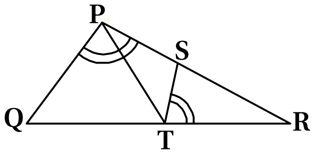
OPEN STUDENT FOUNDATION
STD 10 : MATHS
IMPORTANT QUESTION DAY 5

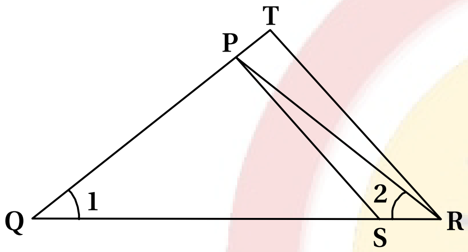
Section A

- **Write the answer of the following questions. [Each carries 4 Marks]** **[48]**

1. S and T are points on sides PR and QR of Δ PQR such that ∠P = ∠RTS. Show that Δ RPQ ~ Δ RTS.



2. In Figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠1 = ∠2. Show that Δ PQS ~ Δ TQR.



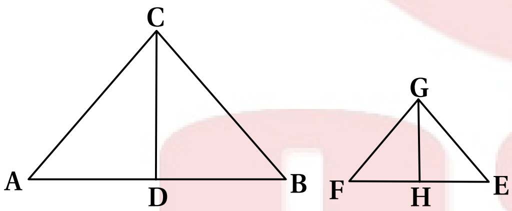
3. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.

4. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If Δ ABC ~ Δ FEG, show that :

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) Δ DCB ~ Δ HGE

(iii) Δ DCA ~ Δ HGF

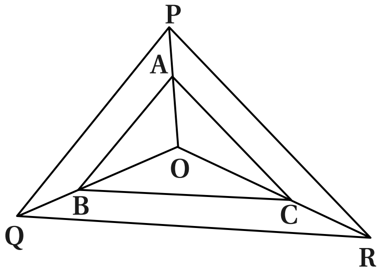


5. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR.

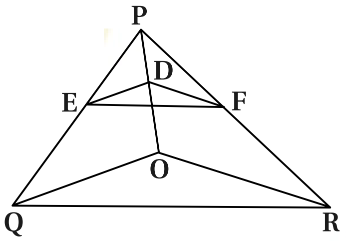
6. If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

7. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show that ABCD is a trapezium.

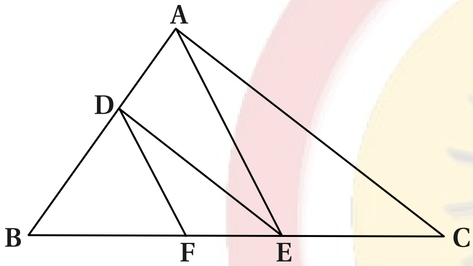
8. In Figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



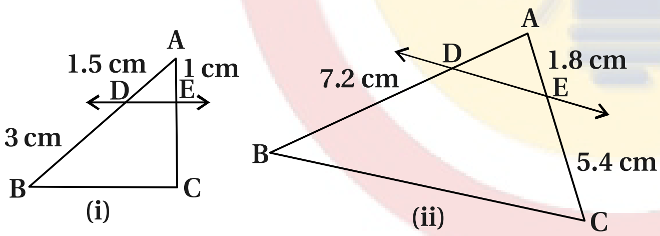
9. In Figure, DE || OQ and DF || OR. Show that EF || QR.



10. In figure, If DE || AC and DF || AE prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



11. In figure (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



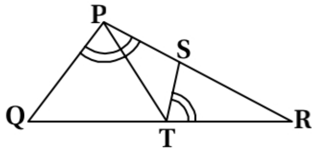
(i) Find EC (ii) Find AD.

12. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Section A

- **Write the answer of the following questions.** [Each carries 4 Marks] **[48]**

1. S and T are points on sides PR and QR of Δ PQR such that ∠P = ∠RTS. Show that Δ RPQ ~ Δ RTS.



➤ In Δ RPQ and Δ RTS,

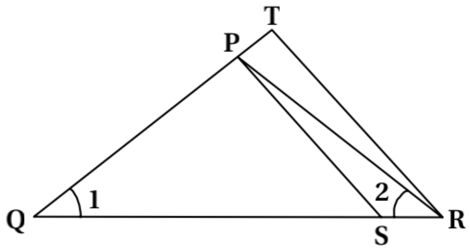
∠RPQ = ∠RTS (given)

∠PRQ = ∠TRS (common angle)

∴ Using AA similarity rule

Δ RPQ ~ Δ RTS.

2. In Figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠1 = ∠2. Show that Δ PQS ~ Δ TQR.



➤ In Δ PQR ∠1 = ∠2 (given)

∴ PR = QP ....(i) (opposite side of equal angle)

➤ $\frac{QR}{QS} = \frac{QT}{PR}$ (given)

∴ $\frac{QR}{QS} = \frac{QT}{QP}$ ....(ii)

➤ In Δ PQS and Δ TQR,

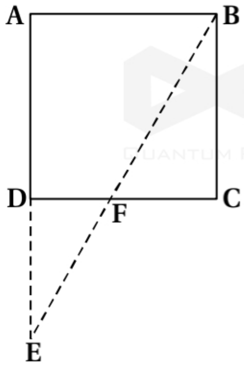
$\frac{QR}{QS} = \frac{QT}{QP}$

$\frac{QS}{QR} = \frac{QP}{QT}$ ....(iii)

∴ ∠SQP = ∠RQT = ∠1

∴ Using SAS similarity rule Δ PQS ~ Δ TQR.

3. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.



➤ In Δ ABE and Δ CFB,

∠BAE = ∠FCB ....(i)

Opposite angle of $\square^m$ are equation)

AB || CD and BE is transversal

∠AEB = ∠CBF ....(ii)

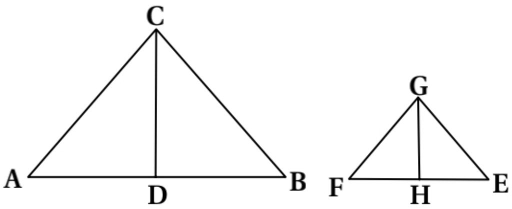
➤ From (i) and (ii) Δ ABE ~ Δ CFB, we have (AA similarity)

4. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If Δ ABC ~ Δ FEG, show that :

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) Δ DCB ~ Δ HGE

(iii) Δ DCA ~ Δ HGF



➤ **(i)** $\mathbf{\frac{CD}{GH} = \frac{AC}{FG}}$

➤ In Δ ACD and Δ FGH,

Δ ABC ~ Δ FGE ∴ ∠A = ∠F

∴ ∠CAD = ∠GFH ...(i)

and ∠C = ∠G

∴ ∠ACD = ∠FGH ....(ii)

➤ From (i) and (ii) (AA similarity)

Δ ACD ~ Δ FGH.

∴ $\frac{CD}{GH} = \frac{AC}{FG}$ (In a similar triangles, the corresponding sides are in proportional.)

(ii) Δ DCB ~ Δ HGE

➤ Δ ABC ~ Δ FEG (given)

∠B = ∠E

∴ ∠DBC = ∠HEG ....(i)

Again Δ ABC ~ Δ FEG

$\therefore$ $\angle$ACB = $\angle$FGE

$\therefore$ $\frac{1}{2}$ $\angle$ACB = $\frac{1}{2}$ $\angle$FGE

$\therefore$ $\angle$ DCB = $\angle$HGE ....(ii)

➤ From (i) and (ii) Δ DCB ~ Δ HGE (AA similarity)

(iii) Δ DCA ~ Δ HGF

➤ Δ ABC ~ Δ FEG

$\therefore$ $\angle$CAB = $\angle$GFE

$\therefore$ $\angle$DAC = $\angle$HFG ....(i)

Again Δ ABC ~ Δ FEG

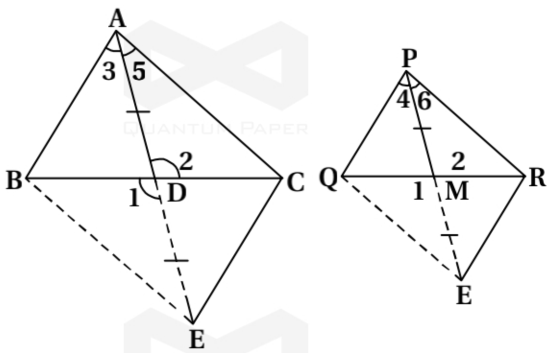
$\therefore$ $\frac{1}{2}$ $\angle$ACB = $\frac{1}{2}$ $\angle$FGE

$\angle$DCA $\cong$ $\angle$HGF ....(ii)

➤ From (i) and (ii)

Δ DCA ~ Δ HGF (AA similarity)

5. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR.

➤ 

➤ **Solution :** We have two Δ ABC and Δ PQR such that AD and PM are medians corresponding to BC and QR respectively.

Produce AD up to point E such that AD = DE and similar produce PM up to N such that PM = MN.

Join EC and NR.

In Δ ADC and Δ EDB,

DC = DB. (D is midpoint by BC)

AD = DE (construction)

$\therefore$ $\angle$ADC = $\angle$BDE (verticle opposite angle)

$\therefore$ Δ ADC $\cong$ ΔEDB (By S.A.S. similarity)

$\therefore$ AC $\cong$ EB... (i) (by C.P.C.T.)

Similarly we can prove Δ PMR $\cong$ Δ NMQ

$\therefore$ PR = NQ ....(ii) (CPCT)

➤ Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\therefore \frac{AB}{PQ} = \frac{EB}{NQ} = \frac{AD}{PM}$ (From (i) and (ii))

$\therefore \frac{AB}{PQ} = \frac{EB}{NQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

$\therefore \Delta ABE \sim \Delta PQN$ (SSS similarity)

$\therefore \angle ABE = \angle PQN$ (C. P. C. Y)

$\therefore \angle 3 = \angle 4$ ....(iii)

Similarly we can prove $\angle 5 = \angle 6$ ....(iv)

➤ From (iii) and (iv)

$\angle 3 + \angle 5 = \angle 4 + \angle 6$

$\therefore \angle A = \angle P$

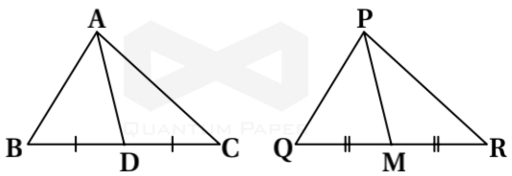
➤ In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle A = \angle P$

$\therefore$ SAS similarity $\Delta ABC \sim \Delta PQR$.

6. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

➤



➤ We have $\Delta ABC \sim \Delta PQR$ such that AD and PM are the median corresponding to the sides BC and QR respectively and the corresponding sides of similar triangles are proportional.

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ....(i)

Corresponding Angle are also equal in two similar triangles.

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ....(ii)

➤ Since AD and PM are medians.

$\therefore BC = 2BD$ and $QR = 2QM$ (iii)

From (i)

$\therefore \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM}$ ....(iii)

And $\angle B = \angle Q \Rightarrow \angle ABD = \angle PQM$ ....(iv)

➤ From (iii) and (iv) we have

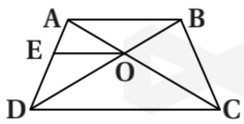
$\Delta ABD \sim \Delta PQM$ (SAS similarity)

Their corresponding sides are proportional

$\therefore \frac{AB}{PQ} = \frac{AD}{PM}$

∵ PQ PM

7. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show that ABCD is a trapezium.

➤ 

➤ We have trapezium ABCD in which diagonals AC and BD Intersect each other at O such that.

$\therefore \frac{AO}{BO} = \frac{CO}{DO}$ (given)

$\therefore \frac{AO}{CO} = \frac{BO}{DO}$

➤ In Δ ADB Draw the EO || AB such that A–E–D and A–O–C.

$\therefore \frac{DE}{EA} = \frac{OD}{BO}$

$\therefore \frac{EA}{DE} = \frac{BO}{DO}$ ....(i)

But $\frac{AO}{CO} = \frac{BO}{DO}$ ....(ii) (given)

➤ From (i) and (ii)

$\frac{EA}{DE} = \frac{BO}{DO} = \frac{AO}{CO}$

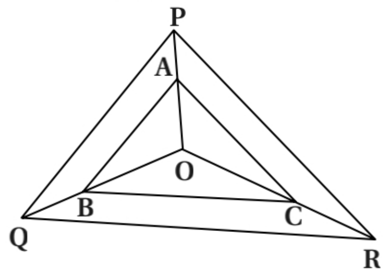
So in Δ ADB, E $\in$ AB and O $\in$ AC (using converse of basic Proportionality theorem)

OE || DC and OE || AB

$\therefore$ AB || DC

$\therefore$ ABCD is a trapezium.

8. In Figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



➤ In Δ OPQ AB || PQ (given)

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ ....(i) (Basic proportionality Theorem)

➤ In Δ OPR AC || PR (given)

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ ....(ii) (Basic proportionality Theorem)

➤ From (i) and (ii) $\frac{OB}{BQ} = \frac{OA}{AP} = \frac{OC}{CR}$

OB OC

$\therefore \frac{\quad}{BQ} = \frac{\quad}{CR}$

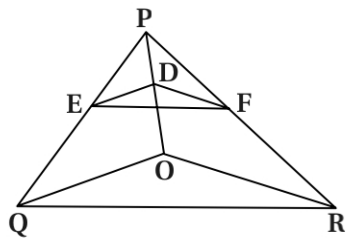
➤ In Δ OQR, $B \in OQ$, $C \in OR$

and $\frac{OB}{BQ} = \frac{OC}{CR}$

Point B and C divided sides OQ and OR in the same ratio.

$\therefore$ BC || QR (Basic proportionality Theorem)

9. In Figure, DE || OQ and DF || OR. Show that EF || QR.



➤ In Δ POQ DE || OQ (given)

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ ....(i) (Basic proportionality Theorem)

➤ In Δ POR DF || OR (given)

$\therefore \frac{PD}{DO} = \frac{PF}{FR}$ ....(ii) (Basic proportionality Theorem)

➤ From (i) and (ii) $\frac{PE}{EQ} = \frac{PD}{DO} = \frac{PF}{FR}$

$\therefore \frac{PE}{EQ} = \frac{PF}{FR}$

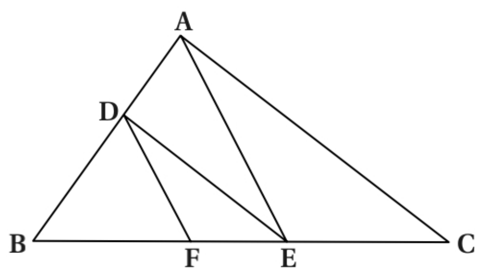
Similarly In Δ POR in which we get EF || QR.

$\therefore \frac{PE}{EQ} = \frac{PE}{FR}$

E and F are dividing the PQ and PR in same ratio.

$\therefore$ EF || QR (Basic proportionality Theorem)

10. In figure, If DE || AC and DF || AE prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



➤ In Δ ABC DE || AC (given)

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ ....(i) (Basic proportionality Theorem)

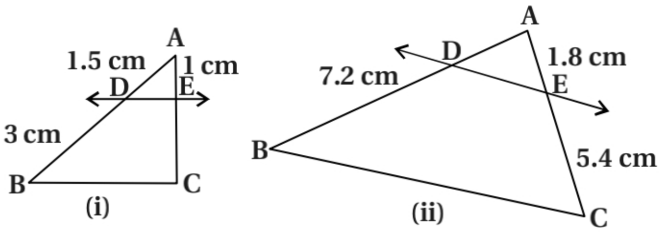
➤ Δ ABE માં DF || AE (given)

$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ ....(ii) (Basic proportionality Theorem)

➤ From (i) and (ii),

$$\frac{BF}{FE} = \frac{BE}{EC}$$

11. In figure (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

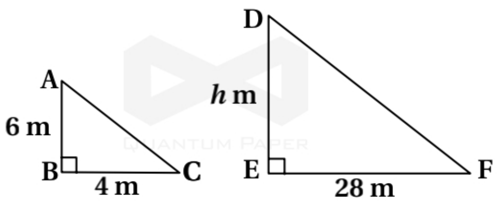


(i) Find EC (ii) Find AD.

➤ In figure DE || BC	DE || BC
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$	$\therefore \frac{AD}{DB} = \frac{AE}{EC}$
$\therefore \frac{1.5}{3} = \frac{1}{EC}$	$\therefore \frac{AD}{7.2} = \frac{1.8}{5.4}$
$\therefore EC \times 1.5 = 3$	$\therefore AD \times 5.4 = 7.2 \times 1.8$
$\therefore EC = \frac{3}{1.5}$	$\therefore AD = \frac{7.2 \times 1.8}{5.4}$
$\therefore EC = 2$ cm	$\therefore AD = \frac{18}{10} \times \frac{72}{10} \times \frac{10}{54}$
	$\therefore AD = 2.4$
	$\therefore AD = 2.4$ cm

12. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

➤



➤ In Δ ABC and Δ DEF,

Length of pole AB = 6 m

Length of shadow of pole BC = 4 m

Length of shadow of tower EF = 28 m

Suppose length of tower DE = h m

In Δ ABC and Δ DEF we have,

$\angle B = \angle E = 90°$

$\angle A = \angle D$ ($\because$ Angular elevation of the sun same time)

Using AA criterion of similarity we have

$\Delta ABC \sim \Delta DEF$

Theirs sides are proportional

$$\therefore \frac{AB}{DE} = \frac{BC}{EF}$$

$$\therefore \frac{6}{h} = \frac{4}{28}$$

$$\frac{6 \times 28}{4} = h = 42 \text{ m}$$

Thus the required height of the tower is 42 m.