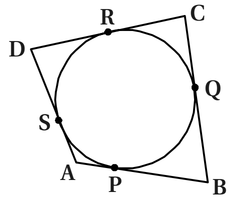
Section A

- **Write the answer of the following questions.** [Each carries 3 Marks] **[21]**

1. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

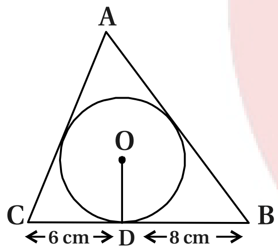
2. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

3. A quadrilateral ABCD isdrawn to circumscribe a circle (see Figure). Prove that AB + CD = AD + BC



4. Prove that the parallelogram circumscribing a circle is a rhombus.

5. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Figure). Find the sides AB and AC.



6. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

7. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

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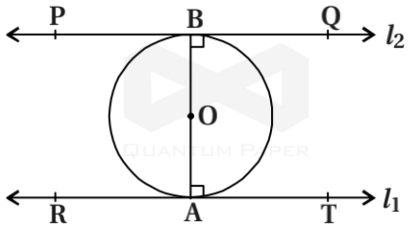
1. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

➤ 

AB is a diameter of a circle with centre O. PQ and RT are tangents drawn at points B and A respectively.

AB is a diameter and BA $\perp$ RT $\Rightarrow \angle RAB = 90°$

Also AB $\perp$ PQ $\Rightarrow \angle ABQ = 90°$

They are alternate angles formed by the line PQ and RT with transversal AB and $\angle RAB = \angle ABQ$

$\therefore$ RT || PQ

2. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

➤ The centre of two

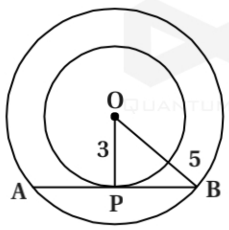
concentric circles is O.

The chord AB of the larger circle touches the smaller circle at the point P.

OP = The radius of the smaller circle = 3 cm.

OB = The radius of the larger circle = 5 cm

AB touches $\odot(0, 3)$ at a point P.



$\therefore$ AB $\perp$ OP

In ΔOPB, $\angle OPB = 90°$ $\therefore$ OB is hypotenuse

$\therefore OB^2 = OP^2 + PB^2$

$\therefore (5)^2 = (3)^2 + PB^2$

$\therefore 25 = 9 + PB^2$

$\therefore PB^2 = 25 - 9 = 16$

$\therefore$ PB = 4 cm

From a centre O of the circle,

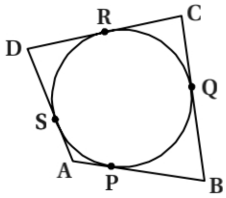
OP $\perp$ AB and AB is a chord of O(0, 5)

∴ P is the midpoint of AB.

∴ The length of the required chord

AB = 2PB = 2 × 4 = 8 cm.

3. A quadrilateral ABCD isdrawn to circumscribe a circle (see Figure). Prove that AB + CD = AD + BC



➤ Let the sides AB, BC, CD and DA of a quadrilateral touch the circle at points P, Q, R and S respectively.

∴ AP = AS, DS = DR, CR = CQ, BQ = BP ....(i)

and A–P–B, B–Q–C, C–R–D and A–S–D ....(ii)

Now AB + CD = AP + PB + CR + RD

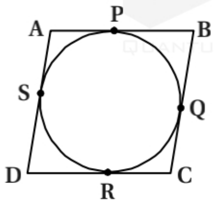
= AS + BQ + CQ + DS

= AS + DS + BQ + CQ

∴ AB + CD = AD + BC

4. Prove that the parallelogram circumscribing a circle is a rhombus.

➤ This sides AB, BC, CD and DA of a parallelogram ABCD touch the circle at the points P, Q, R and S respectively. AP and AS are tangents to the circle from the external point A.



∴ AP = AS ....(i)

similarly BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

Adding the results (i) to (iv) we get

AP + BP + CR + DR = AS + BQ + CQ + DS

∴ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

∴ AB + CD = AD + BC ....(v)

But ☐ ABCD is a parallelogram

∴ AB = CD and BC = AD ....(vi)

From result (v) and (vi) we have

AB + AB = AD + AD

∴ 2AB = 2AD

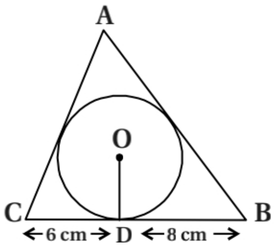
∴ AB = AD (∵ Adjacent sides are equal)

In a parallelogram, AB = AD

$\therefore$ AB = BC = CD = AD

$\therefore$ $\square$ ABCD is a rhombus.

5. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Figure). Find the sides AB and AC.



➤ A Δ ABC is circumscribed in $\odot$ (0, 4).

The sides BC, CA and AB of ΔABC touch the circle at the points D, E and F respectively.

$\therefore$ BF = BD = 8 cm

CE = CD = 6 cm

AF = AE = say x cm

In ΔABC, CB = 14 cm,

CA = $(6 + x)$ cm and

AB = $(x + 8)$ cm.

Perimeter of Δ ABC = AB + BC + CA

$= (x + 8) + (14) + (6 + x)$

$= 2x + 28$

$\therefore$ Half perimeter of ΔABC = S =

$$= \frac{28 + 2x}{2}$$

$\therefore$ By Hero's formula, Area of ΔABC = $\sqrt{S(S - AB)(S - BC)(S - AC)}$

$$= \sqrt{(x+14)(x+14-x-8)(x+14-14)\ (x+14)-6\ -\ x]}$$

$$= \sqrt{(14 + x)(6)(x)(8)}$$

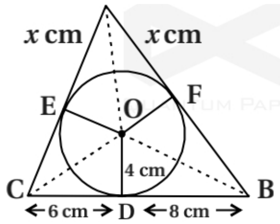
$$= \sqrt{48x\ (x + 14)}\ \text{cm}^2 \quad \text{....(i)}$$

Area of ΔOBC $= \frac{1}{2} \times BC \times OD$

$= \frac{1}{2} \times 14 \times 4$ $\quad (\because$ OD = radius$)$

$= 28\ \text{cm}^2$

A



Area of $\Delta OCA = \frac{1}{2} \times CA \times OE$

$= \frac{1}{2} \times (x + 6) \times 4$

$= 2(x + 6)$

$= (2x + 12)\ cm^2$

Area of $\Delta OAB = \frac{1}{2} \times AB \times OF$

$= \frac{1}{2} \times (x + 8) \times 4$

$= 2(x + 8)$

$= 2x + 16\ cm^2$

Area of ΔABC = Area of ΔOBC ₹ Area of ΔOCA ₹ Area of ΔOAB.

$= 28\ cm^2 + (2x + 12)cm^2 + (2x + 16)\ cm^2$

$= (56 + 4x)\ cm^2$ ....(ii)

From result (i) and (ii), we have.

$56 + 4x = \sqrt{(48x\ (x + 14)}$

$\therefore 4(14 + x) = \sqrt{(14 + x)48x}$

$\therefore 4(14 + x) = 4\sqrt{(14 + x)\ 3x}$

$\therefore 14 + x = \sqrt{(14 + x)\ 3x}$

$\therefore (14 + x)^2 = \left(\sqrt{(14 + x)\ 3x}\right)^2$

$\therefore (14 + x)\ (14 + x) = (14 + x)3x$

$\therefore 14 + x = 3x$

$\therefore 3x - x = 14$

$\therefore 2x = 14$

$\therefore x = 7$

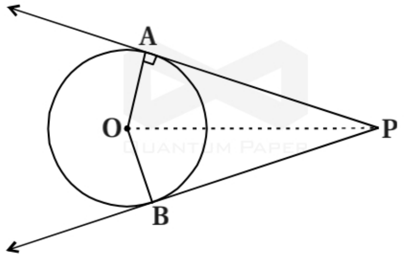
➡ AB = 8 + 7 = 15 cm

➡ BC = 8 + 6 = 14 cm

➡ CA = 6 + 7 = 13 cm

Thus, AB = 15 cm and AC = 13 cm.

6. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

➤ 

Let PA and PB are tangent to the circle with centre O drawn from the external point.

In right angle ΔOAP and ΔOBP,

PA = PB (Tangents with equal lengths)

OA = OB (radii of the same circle)

OP = OP (common)

$\therefore$ By SSS congruence, $\Delta OAP \cong \Delta OBP$

$\therefore$ $\angle OAP = \angle OBP$ and $\angle AOP = \angle BOP$ (CPCT)

$\therefore$ $\angle APB = 2\angle OPA$ and $\angle AOB = 2\angle AOP$

But $\angle AOP = 90° - \angle OPA$

$\therefore$ $2\angle AOP = 180° - 2\angle OPA$ (multiply by 2)

$\therefore$ $\angle AOB = 180° - \angle APB$

$\therefore$ $\angle AOB + \angle APB = 180°$

Hence proved.

7. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

➤ O is the centre of the circle. The sides AB, BC, CD and DA of a quadrilateral touch the circle at P, Q, R and S respectively.

Join OP, OQ, OR and OS. We know that the tangents drawn from an external point of the circle make equal angle at the centre of the circle.

$\therefore$ $\angle 1 = \angle 2$

$\therefore$ $\angle 3 = \angle 4$

$\therefore$ $\angle 5 = \angle 6$

and $\angle 7 = \angle 8$

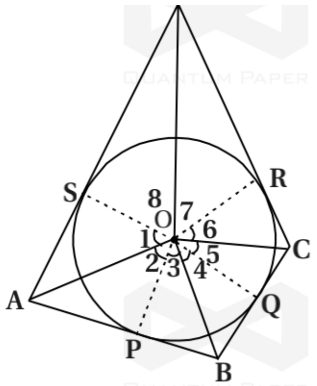
$\angle 2 + \angle 3 = \angle AOB$

$\angle 6 + \angle 7 = \angle COD$

$\angle 1 + \angle 8 = \angle AOD$

$\angle 4 + \angle 5 = \angle BOC$

D



The sum of all the angles at centre is 360°

$\therefore\ \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\therefore\ 2[\angle 1 + \angle 8 + \angle 5 + \angle 4] = 360°$

$\therefore\ \angle 1 + \angle 8 + \angle 5 + \angle 4 = 180°$ ....(i)

and $2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360°$

$\therefore\ \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$ ....(ii)

From results (i) and (ii) we have,

$\angle AOD + \angle BOC = 180°$ and

$\angle AOB + \angle COD = 180°$

Hence proved.