OPEN STUDENT FOUNDATION

CHAPTER 3 — Std 12 : MATHS — Date : 19/02/24

PRACTICE SHEET DAY 2

Section A

- **Choose correct answer from the given options. [Each carries 1 Mark]** **[11]**

1. The number of all possible matrices of order 3×3 with each entry 0 or 1 is :
(A) 27 (B) 18 (C) 81 (D) 512

2. The restriction on n, k and p so that PY + WY will be defined are :
(A) $k = 3,\ p = n$ (B) k is arbitrary, $p = 2$
(C) p is arbitrary, $k = 3$ (D) $k = 2,\ p = 3$

3. If $n = p$, then the order of the matrix 7X – 5Z is :
(A) $p \times 2$ (B) $2 \times n$ (C) $n \times 3$ (D) $p \times n$

4. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to ....
(A) A (B) I – A (C) I (D) 3A

5. If the matrix A is both symmetric and skew symmetric, then.....
(A) A is a diagonal matrix (B) A is a zero matrix
(C) A is a square matrix (D) None of these

6. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then.....
(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$ (C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$

7. If $A \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$ then, A = ...
(A) $\begin{bmatrix} 2 & -1 & 1 \\ 0 & -3 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & -2 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 1 & 0 \\ 2 & -3 & 4 \end{bmatrix}$

8. If $A = [a,\ b]$, $B = [-b,\ -a]$ and $c = \begin{bmatrix} a \\ -a \end{bmatrix}$ then out of the following ...... statement is true.
(A) A = –B (B) A + B = A – B (C) AC = BC (D) CA = CB

9. $\begin{bmatrix} \frac{1}{25} & 0 \\ x & \frac{1}{25} \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-2}$ then x = ...
(A) $\dfrac{a}{125}$ (B) $\dfrac{2a}{25}$ (C) $\dfrac{2a}{125}$ (D) $\dfrac{2a}{25}$

10. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then A = ...
(A) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

11. If $A = \begin{bmatrix} 2x & 9 \\ -3 & -2 \end{bmatrix}$ and $|A| = 3$ then x = ..., $x \in R$.
(A) 7.5 (B) 6 (C) 15 (D) 12

Section B

- **Write the answer of the following questions. [Each carries 2 Marks]** **[10]**

1. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)\ F(y) = F(x + y)$.

2. Express the given matrix as the sum of a symmetric and a skew symmetric matrix : $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

3. For what values of x : $[1\ 2\ 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?

4. Show that the matrix B′AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

5. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Section C

- **Write the answer of the following questions. [Each carries 3 Marks]** **[9]**

6. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

7. Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.

8. Find x, if $[x\ \ -5\ \ -1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$.

Section D

- **Write the answer of the following questions. [Each carries 4 Marks]** **[8]**

9. Find X and Y, if

   (i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

   (ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

10. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ then, prove that $A^3 - 6A^2 + 7A + 2I = O$.

Section A

- **Choose correct answer from the given options. [Each carries 1 Mark]** **[11]**

1. The number of all possible matrices of order 3×3 with each entry 0 or 1 is :

(A) 27 (B) 18 (C) 81 (D) 512

Ans. (D) 512

➠ Here, the order of the matrix is 3×3.

$\therefore$ The number of elements are 9.

Elements are taken 0 and 1.

$\therefore$ The selection of elements is of two ways.

$\therefore$ The number of all possible matrices is $(2)^9 = 512$.

2. The restriction on n, k and p so that PY + WY will be defined are :

(A) $k = 3$, $p = n$ (B) k is arbitrary, $p = 2$

(C) p is arbitrary, $k = 3$ (D) $k = 2$, $p = 3$

Ans. (A) $k = 3$, $p = n$

➥ $X_{2 \times n}$, $Y_{3 \times k}$, $Z_{2 \times p}$, $W_{n \times 3}$ and $P_{p \times k}$ are matrices.

PY + WY will be defined.

$\therefore$ $P_{p \times k} Y_{3 \times k} + W_{n \times 3} Y_{3 \times k}$ will be defined.

$\therefore$ $k = 3$ $[PY]_{p \times k} + [WY]_{n \times k}$ will be defined.

$\therefore$ $p = n$

$\therefore$ (A) is true.

3. If $n = p$, then the order of the matrix $7X - 5Z$ is :

(A) $p \times 2$ (B) $2 \times n$ (C) $n \times 3$ (D) $p \times n$

Ans. (B) $2 \times n$

➥ $7X - 5Z = 7X_{2 \times n} - 5Z_{2 \times p}$

Now $n = p$ is given.

$\therefore$ The order of the matrix.

$\therefore$ $7X - 5Z$ is $2 \times n$.

$\therefore$ (B) is true.

4. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to ....

(A) A (B) I – A (C) I (D) 3A

Ans. (C) I

➥ $A^2 = A \Rightarrow A^3 = A^2$ and $A = I$

$(I + A)^3 - 7A$

$= I^3 + A^3 + 3IA\ (I + A) - 7A$

$= I + A^2 + 3A(I + A) - 7A$

$= I + A^2 + 3A + 3A^2 - 7A$

$= I + 4A^2 - 4A$

$= I + 4A - 4A$ $(\because \text{Take } A^2 = A)$

$= I$

5. If the matrix A is both symmetric and skew symmetric, then.....

(A) A is a diagonal matrix (B) A is a zero matrix

(C) A is a square matrix (D) None of these

Ans. (B) A is a zero matrix

➥ A is both symmetric and skew symmetric matrix.

$\therefore A = A'$ and $A = -A'$

$\therefore A = -A$ $(\because A' = A)$

$\therefore 2A = 0 \Rightarrow A = 0$

$\therefore$ A is a zero matrix.

6. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then.....

(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$ (C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$

Ans. (C) $1 - \alpha^2 - \beta\gamma = 0$

➥ $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$A^2 = I$

$\therefore A \times A = I$

$\therefore \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \alpha^2 + \beta\gamma = 1$ and $\gamma\beta + \alpha^2 = 1$

$\therefore 1 - \alpha^2 - \beta\gamma = 0$

$\therefore$ Alternate (C) is true.

7. If $A \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$ then, A = ...

(A) $\begin{bmatrix} 2 & -1 & 1 \\ 0 & -3 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & -2 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 1 & 0 \\ 2 & -3 & 4 \end{bmatrix}$

Ans. (C) $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$

➥ Suppose, A is $m \times n$ matrix

$\therefore\ (m \times n) \cdot (2 \times 3) = (3 \times 3)$

$\therefore$ It is clear that A is 3×2 matrix.

$\therefore$ A may be option (B) or option (C).

➠ By taking option (B),

$$A\begin{bmatrix}1 & -2 & -5\\3 & 4 & 0\end{bmatrix} = \begin{bmatrix}5 & -2\\1 & 0\\-3 & 4\end{bmatrix}\begin{bmatrix}1 & -2 & -5\\3 & 4 & 0\end{bmatrix}$$

$$= \begin{bmatrix}5-6 & -10-8 & -25\\1+0 & -2 & -5\\-3+12 & 6+16 & 15\end{bmatrix}$$

$$= \begin{bmatrix}-1 & -18 & -25\\1 & -2 & -5\\9 & 22 & 15\end{bmatrix} \neq \begin{bmatrix}-1 & -8 & -10\\1 & -2 & -5\\9 & 22 & 15\end{bmatrix}$$

$\therefore$ Option (B) is not possible.

➠ By taking option (C),

$$A\begin{bmatrix}1 & -2 & -5\\3 & 4 & 0\end{bmatrix} = \begin{bmatrix}2 & -1\\1 & 0\\-3 & 4\end{bmatrix}\begin{bmatrix}1 & -2 & -5\\3 & 4 & 0\end{bmatrix}$$

$$= \begin{bmatrix}2-3 & -4-4 & -10\\1 & -2 & -5\\-3+12 & 6+16 & 15\end{bmatrix}$$

$$= \begin{bmatrix}-1 & -8 & -10\\1 & -2 & -5\\9 & 22 & 15\end{bmatrix}$$

$\therefore$ Option (C) is correct.

8. If $A = [a, b]$, $B = [-b, -a]$ and $c = \begin{bmatrix}a\\-a\end{bmatrix}$ then out of the following ...... statement is true.

(A) $A = -B$ (B) $A + B = A - B$ (C) $AC = BC$ (D) $CA = CB$

Ans. (C) $AC = BC$

➠ $AC = [a^2 - ab]$

$BC = [-ab + a^2]$

$\therefore AC = BC$

$A = [a, b]_{1\times2}$

$B = [-b, -a]_{1\times2}$

$C = \begin{bmatrix}a\\-a\end{bmatrix}_{2\times1}$

9. $\begin{bmatrix}\frac{1}{25} & 0\\x & \frac{1}{25}\end{bmatrix} = \begin{bmatrix}5 & 0\\-a & 5\end{bmatrix}^{-2}$ then $x =$ ....

(A) $\frac{a}{125}$ (B) $\frac{2a}{25}$ (C) $\frac{2a}{125}$ (D) $\frac{2a}{25}$

Ans. (C) $\frac{2a}{125}$

$\Rightarrow \begin{bmatrix} \frac{1}{25} & 0 \\ x & \frac{1}{25} \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-2}$

$\therefore \begin{bmatrix} \frac{1}{25} & 0 \\ x & \frac{1}{25} \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-1}$

$\therefore \begin{bmatrix} \frac{1}{5} & 0 \\ 5x - \frac{a}{25} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-1}$

$\therefore \begin{bmatrix} \frac{1}{5} & 0 \\ 5x - \frac{a}{25} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore 25x - \frac{a}{5} - \frac{a}{5} = 0 \quad \therefore x = \frac{2a}{125}$

10. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then A = ...

(A) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Ans. (C) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$

$\Rightarrow A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$

$= \frac{1}{1} \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \frac{1}{-1} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6-5 & 4-3 \\ -9+10 & -6+6 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

11. If $A = \begin{bmatrix} 2x & 9 \\ -3 & -2 \end{bmatrix}$ and $|A| = 3$ then $x =$ ...., $x \in R$.

(A) 7.5 (B) 6 (C) 15 (D) 12

Ans. (B) 6

$\Rightarrow$ Here $|A| = 3$

$\therefore \begin{vmatrix} 2x & 9 \\ -3 & -2 \end{vmatrix} = 3$

$\therefore 2(-2x) - (9)(-3) = 3$

$\therefore\ -4x + 27 = 3$

$\therefore\ -4x = 3 - 27$

$\therefore\ -4x = -24$

$\therefore\ -x = -6$

$\therefore\ x = 6$

Section B

- **Write the answer of the following questions. [Each carries 2 Marks]** **[10]**

1. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)\ F(y) = F(x + y)$.

➥ $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

L.H.S. $F(x)\ F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x + y) & -\sin(x + y) & 0 \\ \sin(x + y) & \cos(x + y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= F(x + y)$ R.H.S.

Hence $F(x) \cdot F(y) = F(x + y)$ is proved.

2. Express the given matrix as the sum of a symmetric and a skew symmetric matrix : $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

➥ $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ $\therefore A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$\frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$

$$A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$\frac{1}{2}(A + A')$ is a symmetric matrix and $\frac{1}{2}(A - A')$ is a skew symmetric matrix.

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

3. For what values of x : $[1\ 2\ 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?

➥ $[1\ 2\ 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

$\therefore [1 + 4 + 1 \quad 2 + 0 + 0 \quad 0 + 2 + 2]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

$\therefore [6\ \ 2\ \ 4]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

$\therefore 0 + 4 + 4x = O$

$\therefore x = -1$

4. Show that the matrix B′AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

➥ A is symmetric matrix. $\Rightarrow A' = A$

Let $X = B'AB$

$\therefore X' = (B'AB)'$

$= B'A'(B')' \qquad (\because (B')' = B)$

$= B'AB$

$= X$

$\therefore X = B'AB$ is a symmetric matrix.

$\rightarrow$ A is skew symmetric matrix $\Rightarrow A' = -A$

$X' = (B'AB)'$

$= B' A' (B')'$ $(\because (B')' = B)$

$= B'(-A)B$

$= -(B'AB)$

$= -X$

$\therefore$ X is a skew symmetric matrix.

Hence the matrix B'AB is symmetric or skew symmetric according as matrix A is symmetric or skew symmetric.

5. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

➥ A and B are symmetric matrices.

$\therefore A = A'$ and $B = B'$ ....(i)

Let $P = AB - BA$

$\therefore P' = (AB - BA)'$

$= (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$ $(\because$ From (i))

$= -(AB - BA)$

$= -P$

$\therefore$ P = AB – BA is a skew symmetric matrix.

Section C

● **Write the answer of the following questions. [Each carries 3 Marks]** **[9]**

6. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

➥ $A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

Now $A^2 - 5A + 6I$

$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10+6 & -1-0+0 & 2-5+0 \\ 9-10+0 & -2-5+6 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2+0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

7. Find $\frac{1}{2}$ (A + A') and $\frac{1}{2}$(A – A'), when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.

➥ $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \therefore A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$$A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = 2\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \frac{1}{2} \cdot 2\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

8. Find x, if $[x \quad -5 \quad -1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$.

➥ $[x \quad -5 \quad -1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$\therefore [x \quad -5 \quad -1] \begin{bmatrix} 1(x) + 0(4) + 2(1) \\ 0(x) + 2(4) + 1(1) \\ 2(x) + 0(4) + 3(1) \end{bmatrix} = O$

$\therefore [x \quad -5 \quad -1] \begin{bmatrix} x + 2 \\ 8 + 1 \\ 2x + 3 \end{bmatrix} = O$

$\therefore [x \quad -5 \quad -1] \begin{bmatrix} x + 2 \\ 9 \\ 2x + 3 \end{bmatrix} = O$

$\therefore \; x(x + 2) - 45 - 2x - 3 = 0$

$\therefore \; x^2 + 2x - 45 - 2x - 3 = 0$

$\therefore \; x^2 = 48 \; = \pm 4\sqrt{3}$

Section D

- **Write the answer of the following questions. [Each carries 4 Marks]** **[8]**

9. Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

➥ (i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ....(i)

$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ....(ii)

Adding (i) and (ii) $\Rightarrow (X + Y) + (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\therefore 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\therefore X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Sub. (ii) from (i) $\Rightarrow (X + Y) - (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\therefore 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$\therefore Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

➥ (ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ....(i)

$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ ....(ii)

2X (i) − 3(ii) we have,

$$2(2X + 3Y) - 3(3X + 2Y) = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\therefore\ 4X + 6Y - 9X - 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$\therefore\ -5X = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$\therefore\ X = -\frac{1}{5}\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

3X (i) − 2(ii) we have,

$$3(2X + 3Y) - 2(3X + 2Y) = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\therefore\ 6X + 9Y - 6X - 4Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

$$\therefore\ 5Y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

$$\therefore\ Y = \frac{1}{5}\begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

10. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ then, prove that $A^3 - 6A^2 + 7A + 2I = O$.

➥ $$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\text{L.H.S.} = A^3 - 6A^2 + 7A + 2I$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30+7+2 & 0+0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0+0+0+0 & 55-78+21+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{R.H.S.}$$