OPEN STUDENT FOUNDATION
Std 12 : MATHS
PRACTICE SHEET DAY 11

CHAPTER 12 | Date : 29/02/24

Section A

- **Write the answer of the following questions.** [Each carries 3 Marks] **[6]**

1. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 4y$
subject to the constraints : $x + y \le 4$, $x \ge 0$, $y \ge 0$

2. Show that the minimum of Z occurs at more than two points : Maximise $Z = x + y$,
subject to $x - y \le -1$, $-x + y \le 0$, $x, y \ge 0$

Section B

- **Write the answer of the following questions.** [Each carries 4 Marks] **[20]**

3. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 2y$
subject to $x + 2y \le 10$, $3x + y \le 15$, $x, y \ge 0$

4. Solve the following Linear Programming Problem graphically :
Maximise $Z = 5x + 3y$
subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$

5. Show that the minimum of Z occurs at more than two points : Minimise and Maximise $Z = 5x + 10y$
subject to $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$.

6. Show that the minimum of Z occurs at more than two points : Maximise $Z = -x + 2y$, subject to the constraints : $x \ge 3$, $x + y \ge 5$, $x + 2y \ge 6$, $y \ge 0$.

7. Solve the Linear Programming Problem graphically :
Minimise $Z = x + 2y$
subject to $2x + y \ge 3$, $x + 2y \ge 6$, $x, y \ge 0$

Section A

- **Write the answer of the following questions. [Each carries 3 Marks]** **[6]**

1. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 4y$
subject to the constraints : $x + y \le 4$, $x \ge 0$, $y \ge 0$

➥ $x \ge 0,\ y \ge 0$

$\Rightarrow$ The values of x and y are in first quadrant.

$x + y \le 4$

$x + y = 4 \quad \Rightarrow \frac{x}{4} + \frac{y}{4} = 1$

$\Rightarrow$ The line intersects X-axis at A (4, 0) and Y-axis at B(0, 4).

Joining the points A and B, we get the line $x + y = 4$.

For (0, 0), $0 + 0 \le 4$ which is true.

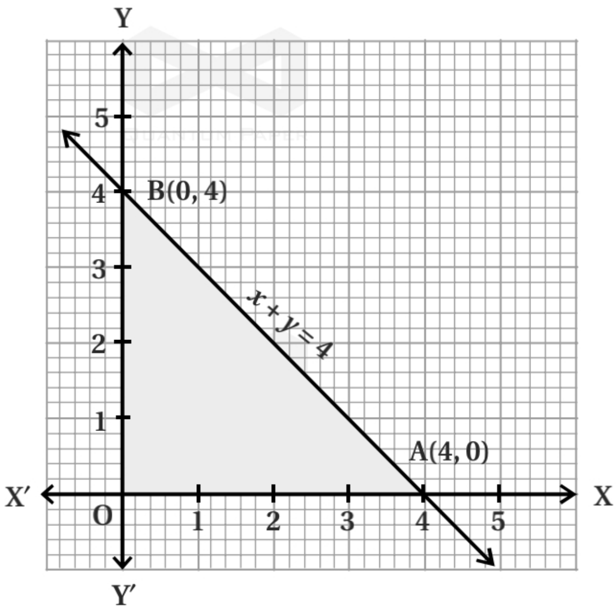
$\therefore$ The half plane containing the origin (0, 0) is the feasible region of the inequality $x + y \le 4$.

Now shaded region OAB is the feasible region of the given linear programming problem. It is bounded region. Its corner points are O(0, 0), A(4, 0) and B(0, 4).

Corner point	Corresponding value of $Z = 3x + 4y$
O(0, 0)	Z = 0
A(4, 0)	Z = 12
B(0, 4)	Z = 16

Hence, the maximum value of $Z = 3x + 4y$ is 16 at a point B(0, 4).

➥



2. Show that the minimum of Z occurs at more than two points : Maximise $Z = x + y$, subject to $x - y \le -1$, $-x + y \le 0$, $x, y \ge 0$

➥ $x, y \geq 0 \Rightarrow$

The values of x and y are in first quadrant.

$x - y \leq -1$

$x - y = -1 \Rightarrow$ The line is passing through A(0, 1) and B(2, 3).

By joining the points A and B, we get the line $x - y = -1$

For (0, 0), $0 - 0 \leq -1$ which is not true.

$\therefore$ The half plane not containing (0, 0) is the solution region of the inequality $x - y \leq -1$.

$-x + y \leq 0$

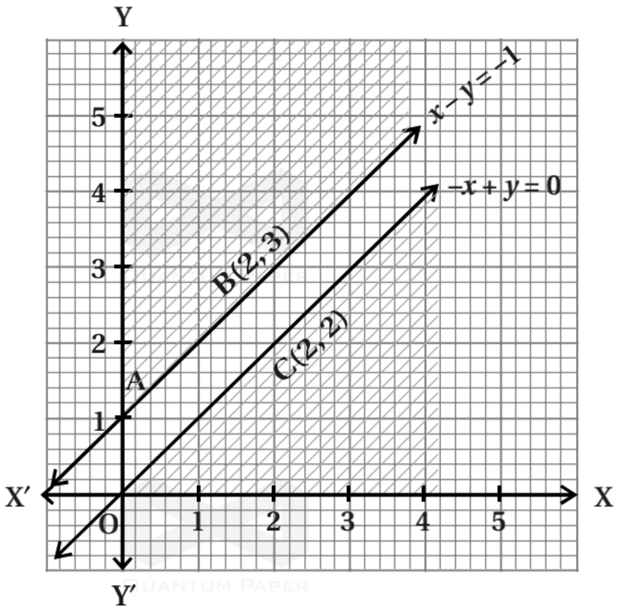
$-x + y = 0 \Rightarrow$ The line is passing through O(0, 0) and C(2, 2)

By joining the points O and C we get the line $-x + y = 0$

Take a point (1, 0). $-1 + 0 \leq 0$ which is true

$\therefore$ The half plane containing (1, 0) is the solution of the region of the inequality $-x + y \leq 0$.

We observe that the two required half planes donot intersect at all i.e. they donot have a common region. Hence there is no maximum Z.



Section B

● **Write the answer of the following questions. [Each carries 4 Marks]** **[20]**

3. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 2y$
subject to $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$

➥ $x, y \geq 0 \Rightarrow$ The values of x and y are in the first quadrant.

$x + 2y \leq 10$

$x + 2y = 10 \Rightarrow \frac{x}{10} + \frac{y}{5} = 1$

$\Rightarrow$ The line $x + 2y = 10$ intersect X-axis at A (10, 0) and Y-axis at B(0, 5)

$x + 2y = 10$ is a line joining the points A(10, 0) and B(0, 5)

For (0, 0), $0 + 0 \leq 10$ which is true.

∴ The half plane of the line $x + 2y = 10$ containing (0, 0) is the solution region of $x + 2y \le 10$

$3x + y \le 15$

$3x + y = 15 \Rightarrow \frac{x}{5} + \frac{y}{15} = 1$

⇒ The line $3x + 5 = 15$ intersects X-axis at A'(5, 0) and Y- axis at B'(0, 15).

$3x + y = 15$ is a line joining the points A'(5, 0) and B'(0, 15)

For (0, 0), $0 + 0 \le 15$ which is true.

∴ The half plane of the line $3x + y = 15$ containing (0, 0) is the solution region of $3x + y \le 15$.

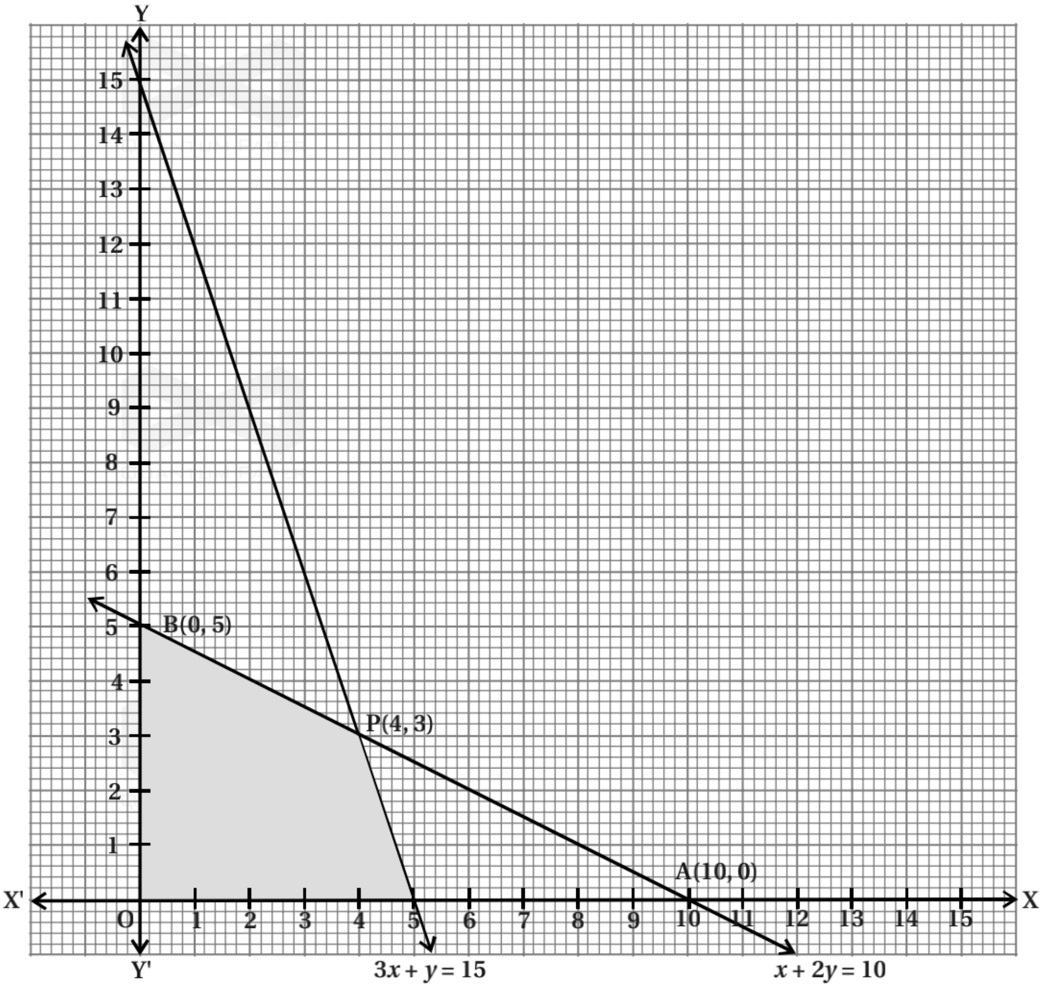
The feasible region of the given linear programming is OA'PB which is shown in figure by shaded region.

This is bounded region. Its corner points are O(0, 0) A'(5, 0) P(4, 3) and B (0, 5).

Corner points	corresponding value of $Z = 3x + 2y$
O(0, 0)	$Z = 0$
A'(5, 0)	$Z = 15$
P(4, 3)	$Z = 18$
B(0, 5)	$Z = 10$

The maximum value of $Z = 3x + 2y$ is 18 at the point P(4, 3) i.e. at $x = 4$ and $y = 3$.

➡



4. Solve the following Linear Programming Problem graphically :

Maximise Z = $5x + 3y$

subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$

➥ $x \geq 0$, $y \geq 0 \Rightarrow$ The values of x and y are in first quadrant.

$3x + 5y = 15 \Rightarrow \frac{x}{5} + \frac{y}{3} = 1$

$\Rightarrow$ The line $3x + 5y = 15$ intersects X-axis at A (5, 0) and Y-axis at B(0, 3).

$3x + 5y = 15$ is a line joining the points A(5, 0) and B(0, 3). For (0, 0), $0 + 0 \leq 15$ which is true.

$\therefore$ The half plane of $3x + 5y = 15$ containing (0, 0) shows the solution region of $3x + 4y \leq 15$

$5x + 2y \leq 10$.

$5x + 2y = 10 \Rightarrow \frac{x}{2} + \frac{y}{5} = 1$

$\Rightarrow$ The line $5x + 2y = 10$ intersects X-axis at A'(2, 0) and Y-axis at B'(0, 5).

$5x + 2y \leq 10$ is a line joining the points

A'(2, 0) and B'(0, 5).

For (0, 0), $0 + 0 \leq 10$ which is true.

$\therefore$ The half plane of $5x + 2y = 10$

containing (0, 0) shows the solution region of $5x + 2y \leq 10$

Hence, the feasible region the given linear programming is OA'PB which is shown as shaded region in graph. By solving the equation,

$3x + 5y = 15$ and $5x + 2y = 10$

We get the co-ordinates of the point P.

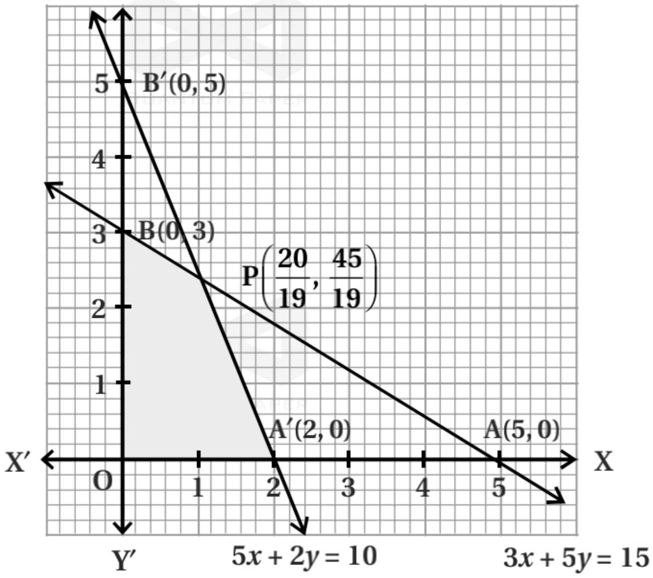
$$\therefore \frac{x}{\begin{vmatrix} 5 & -15 \\ 2 & -10 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -15 & 3 \\ -10 & 5 \end{vmatrix}} = \frac{x}{\begin{vmatrix} 3 & 5 \\ 5 & 2 \end{vmatrix}}$$

$$\Rightarrow x = \frac{20}{19}, \; y = \frac{45}{19}$$

$\therefore$ The co-ordinates of P = $\left(\frac{20}{19}, \frac{45}{19}\right)$

$\therefore$ The corner points of the feasible region are O(0, 0), A'(2, 0), P$\left(\frac{20}{19}, \frac{45}{19}\right)$ and B(0, 3).

➥ Y



Corner points	**Corresponding value of Z = 5x + 3y**
O(0, 0)	Z = 0
A'(2, 0)	Z = 10
$P\left(\frac{20}{19}, \frac{45}{19}\right)$	$Z = \frac{235}{19}$
B(0, 3)	Z = 9

Hence the maximum value of

$Z = 5x + 3y$ is $\frac{235}{19}$ at a point $P\left(\frac{20}{19}, \frac{45}{19}\right)$.

5. Show that the minimum of Z occurs at more than two points : Minimise and Maximise $Z = 5x + 10y$
subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

➥ $x, y \geq 0 \Rightarrow$ The values of x and y are in first quadrant.

$x + 2y \leq 120$

$x + 2y = 120 \Rightarrow \frac{x}{120} + \frac{y}{60} = 1$

$\Rightarrow$ The line intersects X-axis at $A_1(120, 0)$ and Y - axis at $B_1(0, 6)$

$x + 2y = 120$ is a line by joining the points A_1 and B_1

For (0, 0), $0 + 0 \leq 120$ which is true

$\therefore$ The half plane of $x + 2y = 120$ containing (0, 0) is the solution region of $x + 2y \leq 120$

$x + y \geq 60$

$\therefore x + y = 60 \Rightarrow \frac{x}{60} + \frac{y}{60} = 1$

$\Rightarrow$ The line intersects X-axis at A_2 (120, 0) and Y-axis at $B_2(0, 60)$

By joining the points A_2 and B_2 we get the line $x + y = 60$

For (0, 0), $0 + 0 \geq 60$ which is false.

$\therefore$ The half plane not containing (0, 0) is the solution region of $x + y \geq 60$

$x - 2y \geq 0$

$x - 2y = 0 \Rightarrow x = 2y$

$\Rightarrow$ The line is passing through O(0, 0) and A_3 (30, 15)

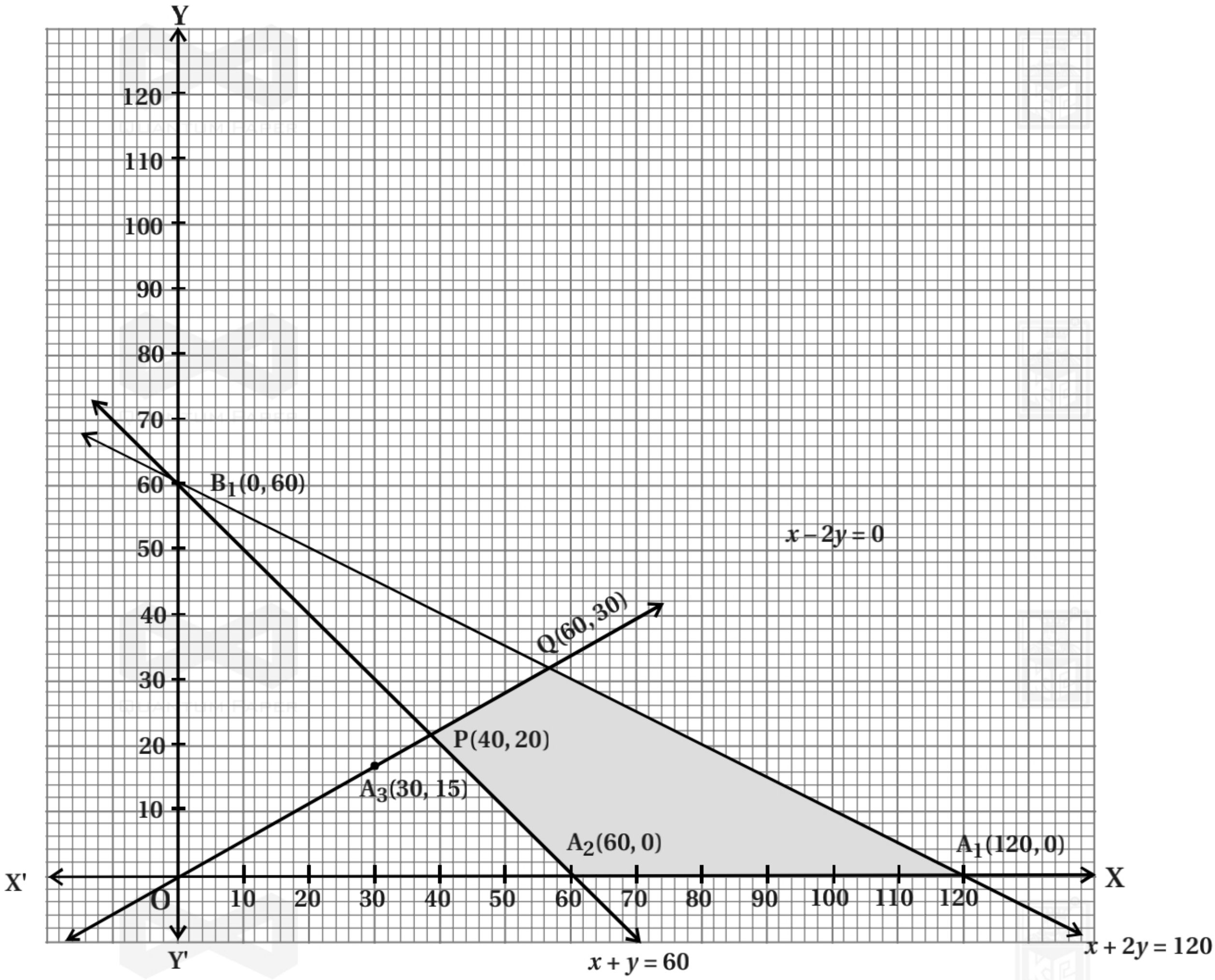
By joining the point O and A_3, we get the line $x - 2y = 0$

Take a point (30, 0)

$\therefore$ 30 – 0 ≥ 0. Which is true.

The half plane containing the point (30, 0) is the solution region of the inequality $x - 2y \geq 0$

The feasible region of the given linear programming is A_1, A_2, PQ, A_1 which is shown in figure by shaded region. It is bounded.



Its corner points are A_1(120, 0) A_2(60, 0) P(40, 20) and Q(60, 30).

Corner points	**corresponding value of Z = 5x + 10y**
A_1(120, 0)	Z = 600
A_2(60, 0)	Z = 300
P(40, 20)	Z = 400
Q(60, 30)	Z = 600

The minimum value of $Z = 5x + 10y$ is 300 at a point A_2(60, 0).

The maximum value of $Z = 5x + 10y$ is 600 at all points on the line segment joining the points A_1(120 , 0) and Q(60, 30).

6. Show that the minimum of Z occurs at more than two points : Maximise $Z = -x + 2y$, subject to the constraints : $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

➥ $x \geq 3$

$x = 3$ is a perpendicular line parallel to Y-axis and passing through (3, 0)

$x \geq 3$ is a region not containing (0, 0) and right side half plane of $x = 3$

$x + y \geq 5$

$x + y = 5 \Rightarrow \frac{x}{5} + \frac{y}{5} = 0$

$\Rightarrow$ The line intersects X-axis at A(5, 0) and Y-axis at B(0, 5).

By joining the points A and B we get the line $x + y = 5$.

For, (0, 0), $0 + 0 \geq 5$ which is not true.

$\therefore$ The half plane not containing (0, 0) is the solution region of the inequality $x + y \geq 5$.

$x + 2y \geq 6$

$x + 2y = 6 \Rightarrow \frac{x}{6} + \frac{y}{3} = 1$

$\Rightarrow$ The line intersects X-axis at C(6, 0) and Y-axis at D(0, 3).

By joining the points C and D, we get the line $x + 2y = 6$.

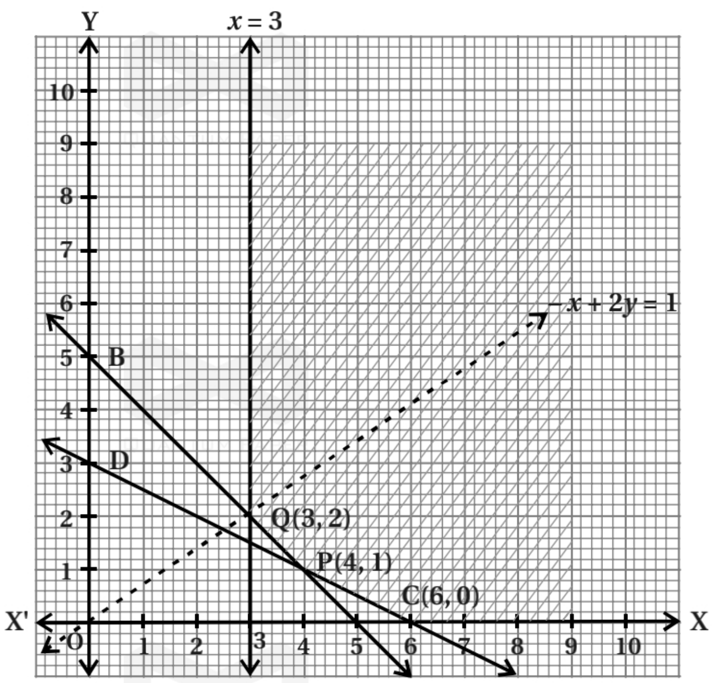
For (0, 0) $0 + 0 \geq 6$ which is not true.

$\therefore$ The half plane not containing (0, 0) is the solution region of the inequality $x + 2y \geq 6$.

The feasible region of the given linear programming is shown in graph. It is unbounded. Its corner points are C(6, 0), P(4, 1) and Q(3, 2).

Corner point	**Corresponding value of $Z = -x + 2y$**
C(6, 0)	$Z = -6$
P(4, 1)	$Z = -2$
Q(3, 2)	$Z = 1$

➥



Y' $x + y = 5$ $x + 2y = 6$

Here the feasible region is unbounded. Therefore 1 may or may not be the maximum value of Z. To decide this, we graph the inequality $-x + 2y > 1$.

There are so many common points in the resulting open half plane with feasible region.

$\therefore$ Z has no maximum value.

7. Solve the Linear Programming Problem graphically :
Minimise $Z = x + 2y$
subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$

➥ $x, y \geq 0 \Rightarrow$ The values of x and y are in the first quadrant

$2x + y \geq 3$

$2x + y = 3 \Rightarrow \frac{x}{\frac{3}{2}} + \frac{y}{3} = 1$

$\Rightarrow$ The line $2x + y = 3$ intersects X- axis at A($\frac{3}{2}$, 0) and B(0, 3).

$2x + y = 3$ is a line joining the points A($\frac{3}{2}$, 0) and B(0, 3) for (0, 0), $0 + 0 \geq 3$ which is false.

$\therefore$ The half plane of $2x + y = 3$ not containing (0, 0) is the solution region of $2x + y \geq 3$

$x + 2y \geq 6$

$x + 2y = 6 \Rightarrow \frac{x}{6} + \frac{y}{3} = 1$

$\Rightarrow$ The line intersects X - axis at A'(6, 0) and Y- axis at B'(0, 3)

$x + 2y = 6$ is a line joining the points A'(6, 0) and B'(0, 3). For (0, 0), $0 + 0 \geq 6$ which is false.
$\therefore$ The half plane of $x + 2y = 6$ not containing (0, 0) is the solution region of $x + 2y \geq 6$.

The feasible region of the given linear programming is XA'BY.

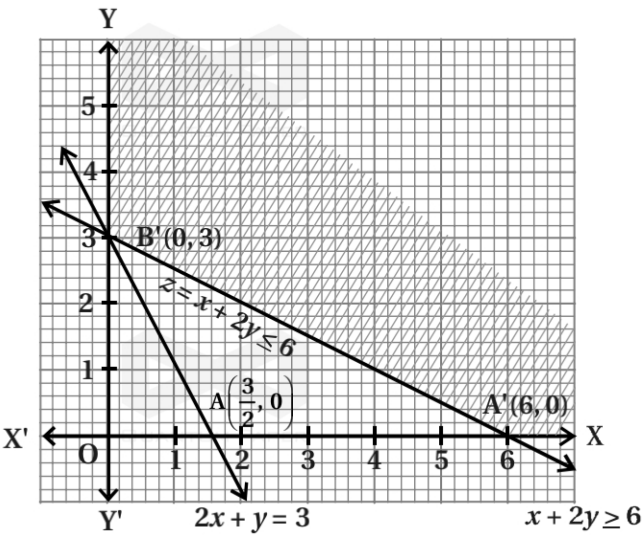
It is unbounded region. Its corner points are A'(6, 0) and B(0, 3)

At A' (6, 0), $Z = x + 2y = 6$

At B(0, 3), $Z = x + 2y = 6$.

The minimum value of Z is 6 at the point A'(6, 0) and B(0, 3).

➥



Here, the minimum value of Z obtain at two points.

But here, the feasible region is unbounded. Therefore, we graph the inequality $x + 2y < 6$.

It is clear that there is no common point between $x + 2y < 6$ and the feasible region.

$\therefore$ The minimum value of Z is 6 at all the points of the line segment joining the points $(6, 0)$ and $(0, 3)$.