OPEN STUDENT FOUNDATION
Std 12 : MATHS
PRACTICE SHEET DAY 12

Date : 22/02/24

Section A

- **Write the answer of the following questions. [Each carries 3 Marks]** **[9]**

1. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 4y$
subject to the constraints : $x + y \le 4$, $x \ge 0$, $y \ge 0$

2. Solve the Linear Programming Problem graphically :
Minimise $Z = -3x + 4y$
subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$

3. Show that the minimum of Z occurs at more than two points : Maximise $Z = x + y$, subject to $x - y \le -1$, $-x + y \le 0$, $x, y \ge 0$

Section B

- **Write the answer of the following questions. [Each carries 4 Marks]** **[16]**

4. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 2y$
subject to $x + 2y \le 10$, $3x + y \le 15$, $x, y \ge 0$

5. Solve the Linear Programming Problem graphically :
Minimise $Z = 3x + 5y$
such that $x + 3y \ge 3$, $x + y \ge 2$, $x, y \ge 0$.

6. Show that the minimum of Z occurs at more than two points : Minimise and Maximise $Z = x + 2y$ subject to $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$, $x, y \ge 0$.

7. Show that the minimum of Z occurs at more than two points : Minimise and Maximise $Z = 5x + 10y$
subject to $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$.

Section A

- **Write the answer of the following questions. [Each carries 3 Marks]** **[9]**

1. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 4y$
subject to the constraints : $x + y \le 4$, $x \ge 0$, $y \ge 0$

➥ $x \ge 0,\ y \ge 0$

$\Rightarrow$ The values of x and y are in first quadrant.

$x + y \le 4$

$x + y = 4 \quad \Rightarrow \dfrac{x}{4} + \dfrac{y}{4} = 1$

$\Rightarrow$ The line intersects X-axis at A (4, 0) and Y-axis at B(0, 4).

Joining the points A and B, we get the line $x + y = 4$.

For (0, 0), $0 + 0 \le 4$ which is true.

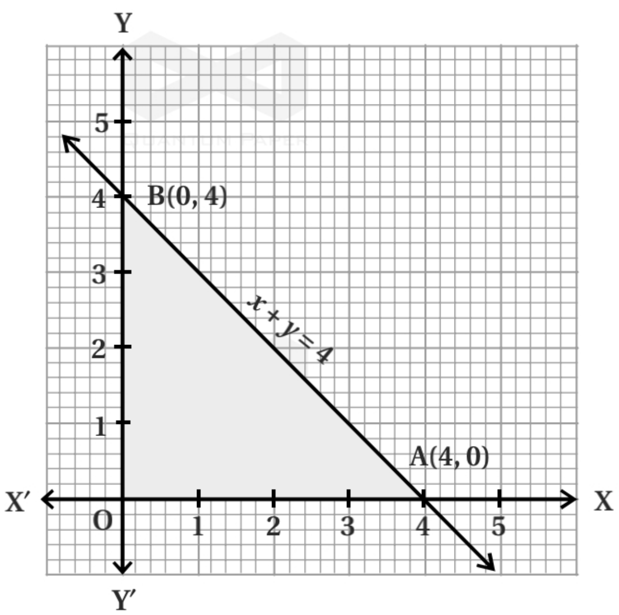
$\therefore$ The half plane containing the origin (0, 0) is the feasible region of the inequality $x + y \le 4$.

Now shaded region OAB is the feasible region of the given linear programming problem. It is bounded region. Its corner points are O(0, 0), A(4, 0) and B(0, 4).

Corner point	Corresponding value of $Z = 3x + 4y$
O(0, 0)	Z = 0
A(4, 0)	Z = 12
B(0, 4)	Z = 16

Hence, the maximum value of $Z = 3x + 4y$ is 16 at a point B(0, 4).

➥



2. Solve the Linear Programming Problem graphically :
Minimise $Z = -3x + 4y$

subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$

➡ $x \ge 0$, $y \ge 0 \Rightarrow$ The values of x and y are in first quadrant.

$x + 2y \le 8$

$x + 2y = 8 \Rightarrow \frac{x}{8} + \frac{y}{4} = 1$

$\Rightarrow$ The line intersects X - axis at A(8, 0) and Y - axis at B(0, 4)

For O(0, 0) $0 + 0 \le 8$ which is true.

$\therefore$ $x + 2y \le 8$ shows the half plane containing the origin O(0, 0)

$3x + 2y \le 12$

$3x + 2y = 12 \Rightarrow \frac{x}{4} + \frac{y}{6} = 1$

$\Rightarrow$ The line intersects X-axis at A'(4, 0) and Y-axis at B'(0, 6).

Again O(0, 0) satisfies $3x + 2y \le 12$

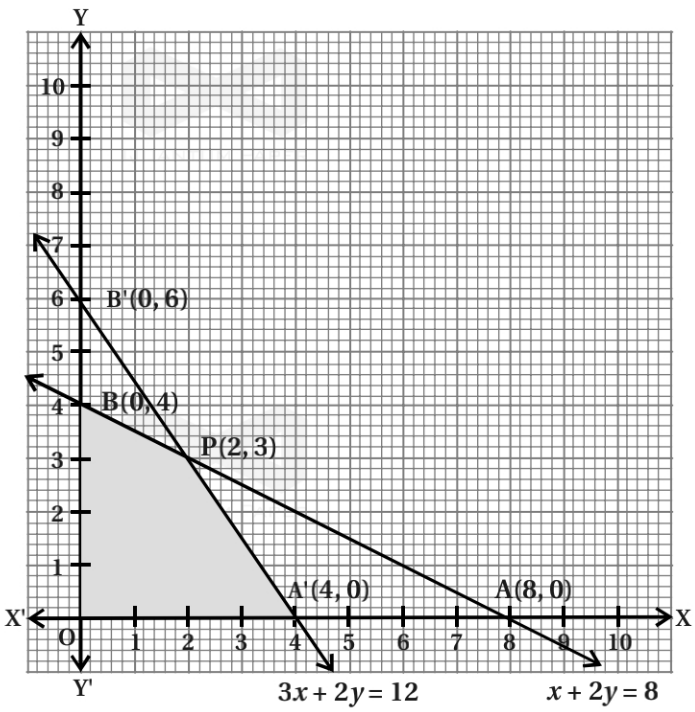
$\therefore$ Half plane containing (0, 0) is the region of $3x + 2y \le 12$

The feasible region is the solution set which is shaded region and is OA'PB in the graph. Its corner points are O(0, 0) A'(4, 0) P(2, 3) and B(0, 4)

Corner point	**Corresponding value of $Z = -3x + 4y$**
O(0, 0)	0
A(4, 0)	–12
P(2, 3)	6
B(0, 4)	16

Hence, the minimum value of $Z = -3x + 4y$ is –12 at the point (4, 0).

➡



3. Show that the minimum of Z occurs at more than two points : Maximise $Z = x + y$, subject to $x - y \le -1$, $-x + y \le 0$, $x, y \ge 0$

➥ $x, y \geq 0 \Rightarrow$

The values of x and y are in first quadrant.

$x - y \leq -1$

$x - y = -1 \Rightarrow$ The line is passing through A(0, 1) and B(2, 3).

By joining the points A and B, we get the line $x - y = -1$

For (0, 0), $0 - 0 \leq -1$ which is not true.

$\therefore$ The half plane not containing (0, 0) is the solution region of the inequality $x - y \leq -1$.

$-x + y \leq 0$

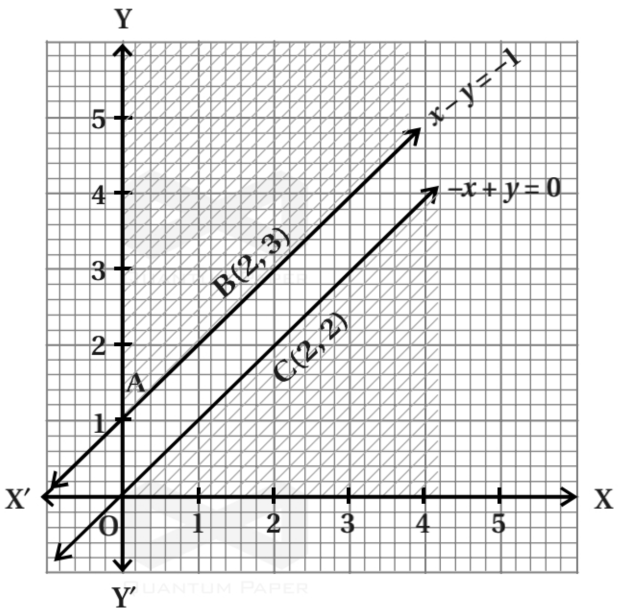
$-x + y = 0 \Rightarrow$ The line is passing through O(0, 0) and C(2, 2)

By joining the points O and C we get the line $-x + y = 0$

Take a point (1, 0). $-1 + 0 \leq 0$ which is true

$\therefore$ The half plane containing (1, 0) is the solution of the region of the inequality $-x + y \leq 0$.

We observe that the two required half planes donot intersect at all i.e. they donot have a common region. Hence there is no maximum Z.



Section B

● **Write the answer of the following questions.** [**Each carries 4 Marks**] **[16]**

4. Solve the Linear Programming Problem graphically :
Maximise $Z = 3x + 2y$
subject to $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$

➥ $x, y \geq 0 \Rightarrow$ The values of x and y are in the first quadrant.

$x + 2y \leq 10$

$x + 2y = 10 \Rightarrow \frac{x}{10} + \frac{y}{5} = 1$

$\Rightarrow$ The line $x + 2y = 10$ intersect X-axis at A (10, 0) and Y-axis at B(0, 5)

$x + 2y = 10$ is a line joining the points A(10, 0) and B(0, 5)

For (0, 0), $0 + 0 \leq 10$ which is true.

∴ The half plane of the line $x + 2y = 10$ containing (0, 0) is the solution region of $x + 2y \le 10$

$3x + y \le 15$

$3x + y = 15 \Rightarrow \dfrac{x}{5} + \dfrac{y}{15} = 1$

$\Rightarrow$ The line $3x + 5 = 15$ intersects X-axis at A'(5, 0) and Y- axis at B'(0, 15).

$3x + y = 15$ is a line joining the points A'(5, 0) and B'(0, 15)

For (0, 0), $0 + 0 \le 15$ which is true.

∴ The half plane of the line $3x + y = 15$ containing (0, 0) is the solution region of $3x + y \le 15$.

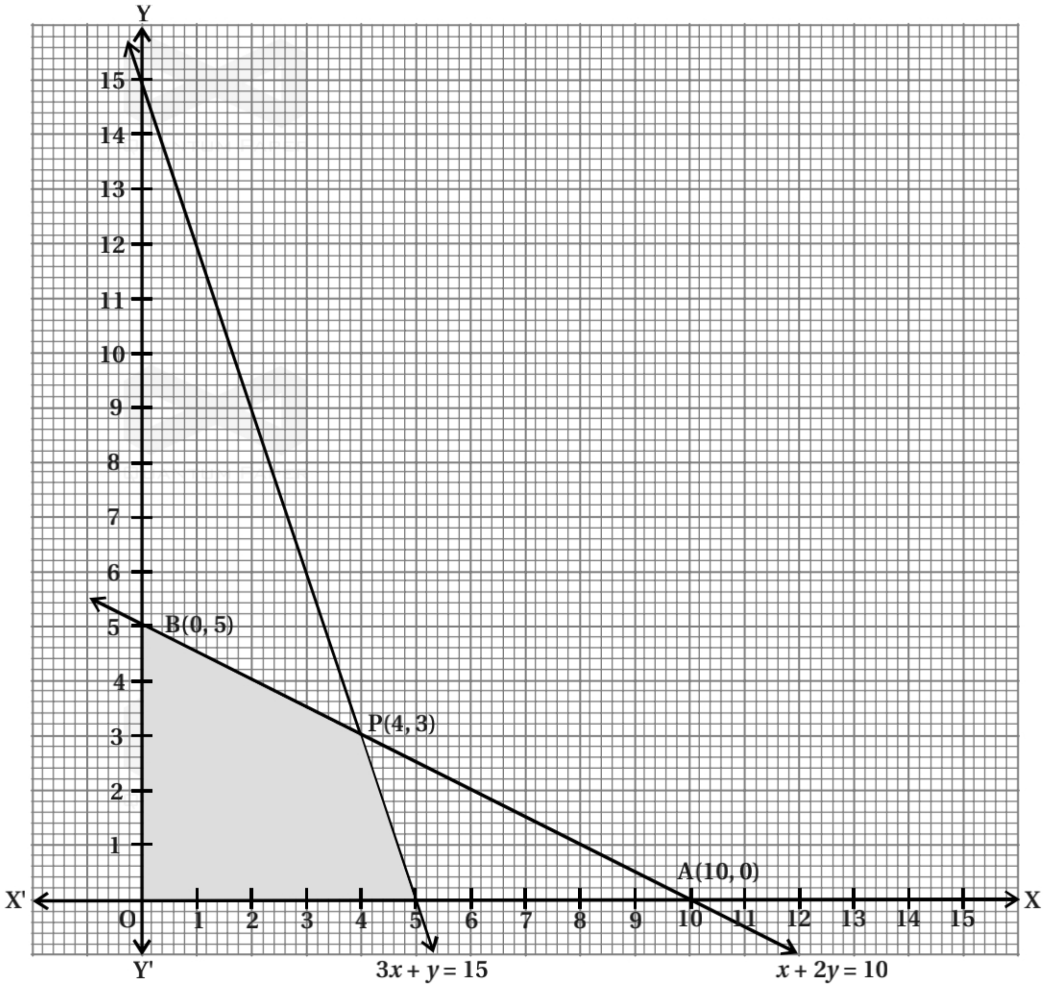
The feasible region of the given linear programming is OA'PB which is shown in figure by shaded region.

This is bounded region. Its corner points are O(0, 0) A'(5, 0) P(4, 3) and B (0, 5).

Corner points	**corresponding value of Z = $3x + 2y$**
O(0, 0)	Z = 0
A'(5, 0)	Z = 15
P(4, 3)	Z = 18
B(0, 5)	Z = 10

The maximum value of $Z = 3x + 2y$ is 18 at the point P(4, 3) i.e. at $x = 4$ and $y = 3$.

➥



5. Solve the Linear Programming Problem graphically :

Minimise Z = $3x + 5y$
such that $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

➥ $x, y \geq 0 \Rightarrow$ The values of x and y are in the first quadrants. $x + 3y \geq 3$

$x + 3y = 3 \Rightarrow \frac{x}{3} + \frac{y}{1} = 1$

$\Rightarrow$ The line $x + 3y = 3$ intersects X-axis at A(3, 0) and Y-axis at B(0, 1).

$x + 3y = 3$ is a line joining the points A(3, 0) and B(0, 1)

For (0, 0), $0 + 0 \geq 3$ which is not true.

$\therefore$ The half plane of $x + 3y = 3$ which does not contain (0, 0) is the solution region of

$x + 3y \geq 3$

$x + y \geq 2$

$x + y = 2 \Rightarrow \frac{x}{2} + \frac{y}{2} = 1$

$\Rightarrow$ The line $x + y = 2$ intersect X-axis at A'(2, 0) and Y-axis at B'(0, 2).

$\therefore$ $x + y = 2$ is a line joining the points A'(2, 0) and B'(0, 2).

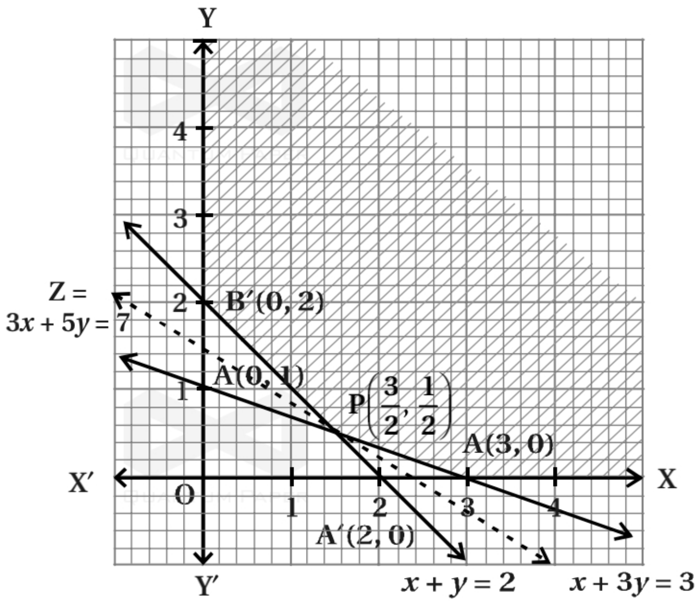
For (0, 0), $0 + 0 \geq 2$ which is not true.

$\therefore$ The half plane of the line $x + y = 2$ which does not contain (0, 0) is the solution region of $x + y = 2$ co-ordinates of point P can be obtained by solving $x + 3y = 3$ and $x + y = 2$ and it is P $\left(\frac{3}{2}, \frac{1}{2}\right)$. The feasible region represented by the given in equalities is APB' which is not bounded.

The corner point of the feasible region are A(3, 0), P$\left(\frac{3}{2}, \frac{1}{2}\right)$ and B'(0, 2).

Corner point	**Corresponding value of Z = $3x + 5y$**
A(3, 0)	Z = 9
P$\left(\frac{3}{2}, \frac{1}{2}\right)$	Z = 7
B'(0, 2)	Z = 10

➥



The minimum value of Z = $3x + 5y$ is 7. But here, the feasible region is unbounded. Therefore, 7 may or may not be the minimum value of Z. To decide this, we graph the inequality $3x + 5y < 7$. $3x + 5y = 7$ is a dotted line as shown in the figure. For (0, 0), $0 + 0 < 7$ containing (0, 0) is the solution region of $3x + 5y < 7$. Now $3x + 5y < 7$ has not common point with the feasible region.

$\therefore$ The minimum value of $Z = 3x + 5y$ is 7 at the point $P\left(\frac{3}{2}, \frac{1}{2}\right)$.

6. Show that the minimum of Z occurs at more than two points : Minimise and Maximise $Z = x + 2y$ subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$.

➡ $x, y \geq 100 \Rightarrow$ The values of x and y are in the first quadrant

$x + 2y \geq 100$

$x + 2y = 100 \Rightarrow \frac{x}{100} + \frac{y}{50} = 1$

$\Rightarrow$ The line intersects X-axis at A(100, 0) and Y- aixs at B(0, 50)

By joining the points A and B, we get the line $x + 2y = 100$. For O(0, 0) $0 + 0 \geq 100$ which is false.

$\therefore$ The half plane not containing (0, 0) is the solution region of $x + 2y \geq 100$

$2x - y \leq 0$

$\therefore$ $2x - y = 0 \Rightarrow 2x = y$

$\Rightarrow$ The line is passing through O(0, 0) and C(50, 100).

Take a point (10, 0)

$\therefore$ $20 - 0 \leq 0$ which is not true.

$\therefore$ The half plane not containing the point (10, 0) is the solution region of $2x - y \leq 0$

$2x + y \leq 200$

$2x + y = 200 \Rightarrow \frac{x}{100} + \frac{y}{200} = 1$

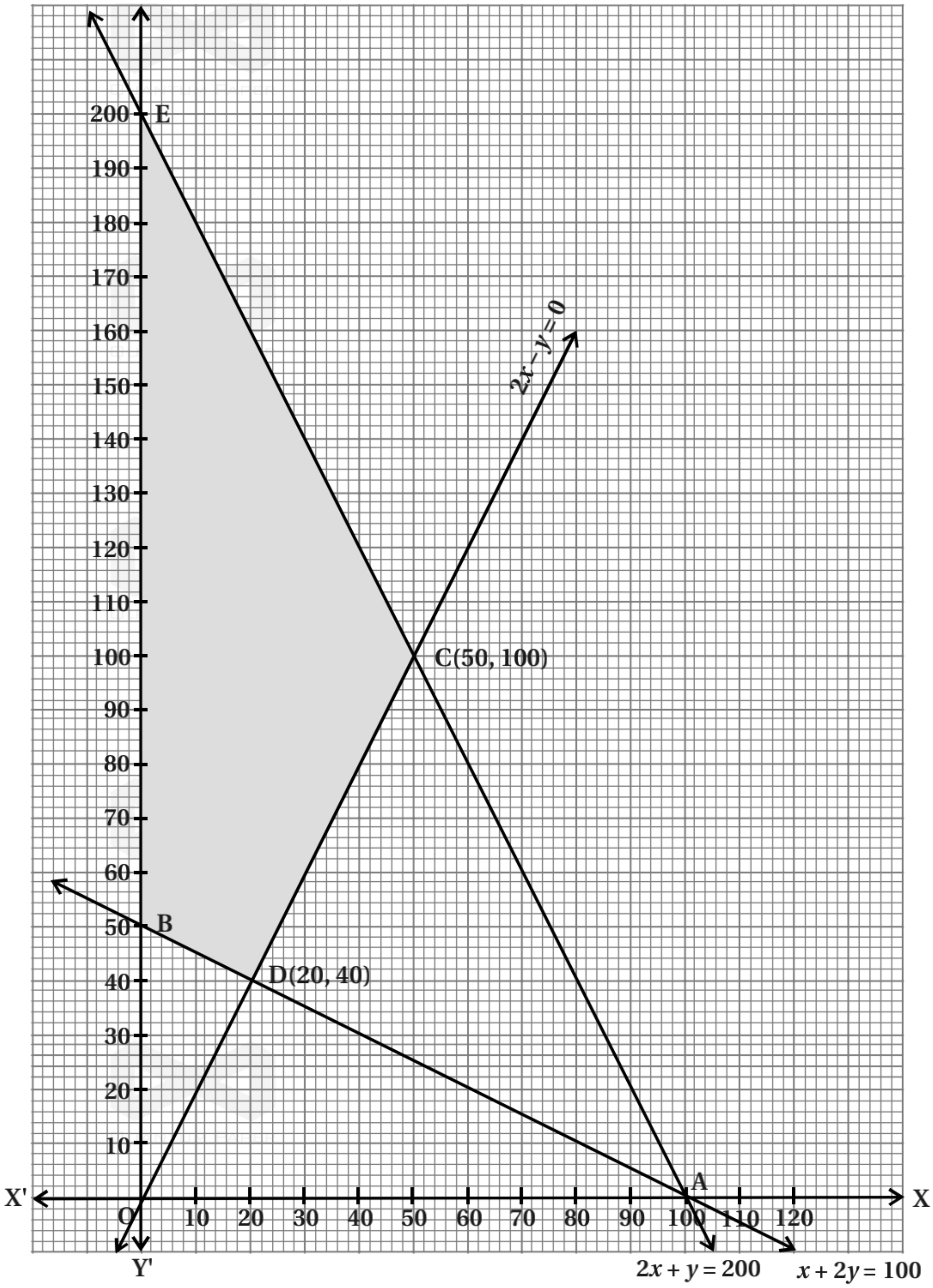
$\Rightarrow$ The line is passing through A(100, 0) and E(0, 200)

By joining the points A(100, 0) and E(0, 200) we get a line $2x + y = 200$

For (0, 0), $0 + 0 \leq 200$ which is true.

$\therefore$ The half plane containing (0, 0) is the solution region of the inequality $2x + y \leq 200$. The feasible region of the given linear programming is BDCEB which is shown by shaded region in the graph. It is the bounded region. Its corner points are B(0, 50), D(20, 40), C(50, 100) and E(0, 200).

➡ Y



Corner points	Corresponding value of Z = $x + 2y$
B(0, 50)	Z = 100
D(20, 40)	Z = 100
C(50, 100)	Z = 250
E(0, 200)	Z = 400

The maximum value of Z = $x + 2y$ is 400 at E(0, 200).

The minimum value of Z = $x + 2y$ is 100 at all the point on the line segment joining the points (0, 50) and (20, 40).

7. Show that the minimum of Z occurs at more than two points : Minimise and Maximise Z = $5x + 10y$ subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

➥ $x, y \geq 0 \Rightarrow$ The values of x and y are in first quadrant.

$x + 2y \leq 120$

$x + 2y = 120 \Rightarrow \frac{x}{120} + \frac{y}{60} = 1$

$\Rightarrow$ The line intersects X-axis at $A_1(120, 0)$ and Y - axis at $B_1(0, 6)$

$x + 2y = 120$ is a line by joining the points A_1 and B_1

For (0, 0), $0 + 0 \leq 120$ which is true

$\therefore$ The half plane of $x + 2y = 120$ containing (0, 0) is the solution region of $x + 2y \leq 120$

$x + y \geq 60$

$\therefore\ x + y = 60 \Rightarrow \frac{x}{60} + \frac{y}{60} = 1$

$\Rightarrow$ The line intersects X-axis at A_2 (120, 0) and Y-axis at $B_2(0, 60)$

By joining the points A_2 and B_2 we get the line $x + y = 60$

For (0, 0), $0 + 0 \geq 60$ which is false.

$\therefore$ The half plane not containing (0, 0) is the solution region of $x + y \geq 60$

$x - 2y \geq 0$

$x - 2y = 0 \Rightarrow x = 2y$

$\Rightarrow$ The line is passing through O(0, 0) and A_3 (30, 15)

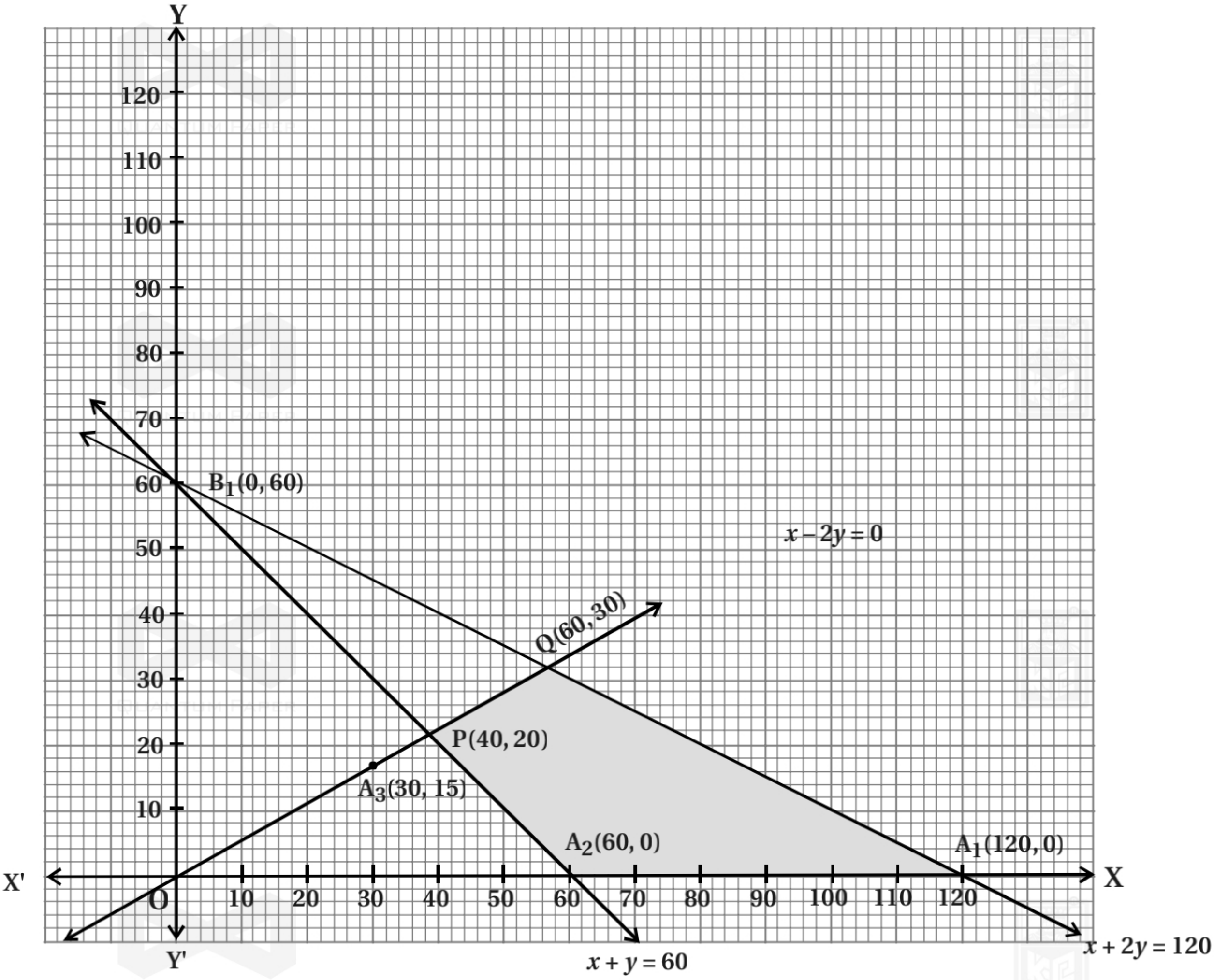
By joining the point O and A_3, we get the line $x - 2y = 0$

Take a point (30, 0)

$\therefore$ $30 - 0 \geq 0$. Which is true.

The half plane containing the point (30, 0) is the solution region of the inequality $x - 2y \geq 0$

The feasible region of the given linear programming is A_1, A_2, PQ, A_1 which is shown in figure by shaded region. It is bounded.

➥ 

Its corner points are $A_1(120, 0)$ $A_2(60, 0)$ P(40, 20) and Q(60, 30).

Corner points	**corresponding value of Z = 5x + 10y**
$A_1(120, 0)$	Z = 600
$A_2(60, 0)$	Z = 300
P(40, 20)	Z = 400
Q(60, 30)	Z = 600

The minimum value of $Z = 5x + 10y$ is 300 at a point $A_2(60, 0)$.

The maximum value of $Z = 5x + 10y$ is 600 at all points on the line segment joining the points A_1(120 , 0) and Q(60, 30).